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The supergravity fields for a D-brane with a travelling wave from string amplitudes

William Black, Rodolfo Russo, David Turton *

Queen Mary University of London, Centre for Research in String Theory, Department of Physics, Mile End Road, London E1 4NS, UK

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ABSTRACT

We calculate the supergravity fields sourced by a D-brane with a null travelling wave from disk amplitudes in type IIB string theory compactified on $T^4 \times S^1$. The amplitudes reproduce all the non-trivial features of the previously known two-charge supergravity solutions in the D-brane/momentum duality frame, providing a direct link between the microscopic bound states and their macroscopic descriptions.

1. Introduction and discussion

D-branes in string theory appear both as classical solutions of the supergravity low-energy effective action and as fundamental objects on which open strings can end. It is well known that mixed open/closed string amplitudes can be used to derive information about the classical solution from the microscopic description [1,2]. In particular, the one-point functions of closed string states from the disk provide a direct way to compute, in the small coupling regime, the backreaction of the D-branes considered [3–6].

Recently it has been shown for two-charge D1–D5 configurations that the same direct link between the microscopic and the macroscopic descriptions holds [7]. Two-charge D-brane configurations have attracted much attention as they represent a simple and tractable system within which (small) black holes in string theory may be studied [8–16]. A natural question is whether these studies can be extended to three-charge D1–D5–P configurations [17,18], since the supergravity description of these configurations is that of a macroscopic black hole. As a further step towards this goal, here we study two-charge configurations involving a momentum charge.

In this Letter we calculate the supergravity fields sourced by a D-brane carrying momentum charge in the form of a null right (or left) moving wave, and show that the fields sourced by this bound state reproduce the non-trivial features of the supergravity solutions which are U-dual to the fundamental string solution of [19,20]. In particular we describe in detail the calculation in the D5–P duality frame. The world-sheet calculation employs the fact that these D-brane configurations admit an exact CFT description [21] in which the travelling wave on the D-brane can be included in the world-sheet action for the open strings in a tractable way. We use the boundary state describing a D-brane with a travelling wave [22–24] to compute the disk one-point functions for emission of massless closed string states, and we read off the various supergravity fields. Contrary to what happens in the D1–D5 frame [7], the string computation in these duality frames yields the full integrals over the D-brane profile appearing in the classical solutions. This is possible because the profile function parametrizing the solutions arises as a condensate of massless open strings related to the physical shape of the D-brane, which can be included exactly in the string world-sheet action.

The direct link between microscopic D-brane configurations and supergravity solutions might also shed further light on the entropy of two charge systems in string theory. It was recently proposed [15] that the macroscopic entropy of a two-charge configuration should be defined to be the sum of the contributions of small black hole solutions and horizonless smooth classical solutions. In this language the term ‘smooth classical solutions’ does not include solutions which are singular due to delta-function sources, and the scaling arguments of [15] applied to the D-brane/momentum duality frame show that α'-corrections to the supergravity action cannot produce small black holes with a non-zero horizon area.

Here we observe that the supergravity solutions which are sourced by the microscopic D-brane bound states are necessarily

* Corresponding author.
E-mail addresses: w.black@qmul.ac.uk (W. Black), r.russo@qmul.ac.uk (R. Russo), d.j.turton@qmul.ac.uk (D. Turton).

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singular at the two-derivative level: the one-point functions on the disk discussed in this Letter provide the asymptotic behaviour of the solutions, and the nonlinear part of the standard supergravity equations of motion determines the background in the interior, leading to the singular backgrounds obtained by dualising the fundamental string solution. Of course, it might still be possible to recover a fully smooth field configuration starting from the same data provided by the disk one-point functions if one includes α'-corrections to the supergravity equations of motion.

This Letter is organised as follows. In Section 2 we review the two-charge supergravity solutions in the D1–P and D5–P duality frames and present the terms in the perturbative expansion that we reproduce from the string amplitudes. In Section 3 we derive the boundary state for these D-brane configurations, compute the one-point functions of the massless closed string states and read off the gravitational backreaction, at weak coupling, of the D-brane configuration.

2. Two-charge system in the D1–P and D5–P duality frames

We work in type IIB string theory on $\mathbb{R}^{4,1} \times S^1 \times T^4$ using the light-cone coordinates $u = (t + y)$, $v = (t - y)$ constructed from the time and S^1 directions. The indices $(I, J, \ldots)$ refer collectively to the other eight directions which we then split into the $\mathbb{R}^4$ directions labelled by $(i, j, \ldots)$ and those along the T^4 labelled by $(a, b, \ldots)$.

The family of classical supergravity solutions in which we are interested describe two-charge D-brane bound states [10,11,25–27] and are connected through T and S dualities to the solution describing a multi-wound fundamental string with a purely right (or left) moving wave [19,20], smeared along the T^4 directions and along y [10,11]. In the D1–P duality frame, we take the D1-brane to be wrapped n_w times around y; letting the length of the y direction be $2\pi R$, the brane then has overall extent $L_T = 2\pi n_w R$ and we use $\hat{v}$ for the corresponding world-volume coordinate on the D-brane, having periodicity L_T. The non-trivial fields are the metric, the dilaton and the R–R 2-form gauge potential:

$$
\begin{aligned}
&ds^2 = H^{-\frac{1}{2}}\, dv \left(-du + K\, dv + 2A_I\, dx^I\right) + H^{\frac{1}{2}}\, dx^I\, dx^I, \\
&e^{2\Phi} = g_s^2 H, \qquad C^{(2)}_{uv} = -\frac{1}{2}\left(H^{-1} - 1\right), \\
&C^{(2)}_{vI} = -H^{-1} A_I,
\end{aligned}
\tag{2.1}
$$

where the harmonic functions take the form

$$
\begin{aligned}
&H = 1 + \frac{Q_1}{L_T}\int_0^{L_T} \frac{d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \qquad A_I = -\frac{Q_1}{L_T}\int_0^{L_T} \frac{d\hat{v}\, \dot{f}_I(\hat{v})}{|x_i - f_i(\hat{v})|^2}, \\
&K = \frac{Q_1}{L_T}\int_0^{L_T} \frac{d\hat{v}\, |\dot{f}_I(\hat{v})|^2}{|x_i - f_i(\hat{v})|^2},
\end{aligned}
\tag{2.2}
$$

where $f_i(\hat{v} + L_T) = f_i(\hat{v})$ and where $\dot{f}$ denotes the derivative of f with respect to $\hat{v}$. The functions f_I describe classically the null travelling wave on the D-string. Q_1 is proportional to g_s and to the D-brane winding number n_w and is given by

$$
Q_1 = \frac{(2\pi)^4 n_w g_s (\alpha')^3}{V_4}.
\tag{2.3}
$$

T-dualising to the D5–P duality frame and using the symmetry of the IIB equations of motion to reverse the sign of B and $C^{(4)}$, we obtain the fields

$$
\begin{aligned}
&ds^2 = H^{-\frac{1}{2}}\, dv \left(-du + \left(K - H^{-1}|A_a|^2\right) dv + 2A_i\, dx_i\right) \\
&\qquad + H^{\frac{1}{2}}\, dx^i\, dx^i + H^{-\frac{1}{2}}\, dx^a\, dx^a, \\
&e^{2\Phi} = \left(g_s'\right)^2 H^{-1}, \qquad B_{va} = -H^{-1} A_a, \\
&C^{(4)}_{vbcd} = -H^{-1} A_a \epsilon_{abcd}, \qquad C^{(6)}_{vi5678} = -H^{-1} A_i, \\
&C^{(6)}_{uv5678} = -\frac{1}{2}\left(H^{-1} - 1\right),
\end{aligned}
\tag{2.4}
$$

where g_s' is the string coupling in the new duality frame and ϵ_{abcd} is the alternating symbol with $\epsilon_{5678} = 1$. The effect of rewriting the functions in (2.2) in terms of D5–P frame quantities is to substitute the D1 with the D5 charge, $Q_1 \to Q_5 = g_s' n_w \alpha'$. From now on, we drop the prime and refer to the D5–P frame string coupling as g_s.

From the large distance behaviour of the g_{vv} component of the metrics above, one can read off how the momentum charge is related to the D-brane profile function f. For instance, in the D1–P frame we have

$$
\frac{n_w}{L_T}\int_0^{L_T} |\dot{f}|^2\, d\hat{v} = \frac{g_s n_p \alpha'}{R^2},
\tag{2.5}
$$

where n_p is the Kaluza–Klein integer specifying the momentum along the compact y direction. From a statistical point of view [13], the typical two-charge bound state with fixed D1 and momentum charges has a profile f consisting of Fourier modes of average frequency $\sqrt{n_w n_p}$. Then (2.5) implies that the typical profile wave has an amplitude of order $\sqrt{g_s}$. Despite this potential g_s dependence, we always keep track of f exactly and expand in the D-brane charges Q_i. From the point of view of the string amplitudes, this means that we are resumming all diagrams with open string insertions describing the D-brane profile, but that we are considering only the disk level contribution.

From now on, for concreteness we present the calculation in the D5–P frame and we focus on the field components that vanish in the absence of a wave; the calculations of the remaining components are analogous. We canonically normalize the metric, B-field and R–R fields:

$$
g = \eta + 2\kappa \hat{h}, \qquad B = \sqrt{2}\kappa \hat{b}, \qquad C = \sqrt{2}\kappa \hat{C}
\tag{2.6}
$$

where as usual, $\kappa = 2^3 \pi^{7/2} g_s (\alpha')^2$. We then expand the relevant components of (2.4) for small Q_5, keeping only linear order terms, which yields the field components that we shall reproduce from the disk amplitudes:

$$
\begin{aligned}
&\hat{h}_{vi} = \frac{Q_5}{2\kappa L_T}\int_0^{L_T} \frac{-\dot{f}_i\, d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \qquad \hat{h}_{vv} = \frac{Q_5}{2\kappa L_T}\int_0^{L_T} \frac{|\dot{f}|^2\, d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \\
&\hat{b}_{va} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T} \frac{\dot{f}_a\, d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \\
&\hat{C}^{(4)}_{vbcd} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T} d\hat{v}\, \frac{\dot{f}_a \epsilon_{abcd}}{|x_i - f_i(\hat{v})|^2}, \\
&\hat{C}^{(6)}_{vi5678} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T} d\hat{v}\, \frac{\dot{f}_i}{|x_i - f_i(\hat{v})|^2}.
\end{aligned}
\tag{2.7}
$$

Similar expressions are easily derived in the D1–P frame from (2.1).

3. Classical fields from string amplitudes

3.1. World-sheet boundary conditions

The key ingredients of our string computation are the boundary conditions which must be imposed upon the world-sheet fields of a string ending on a D-brane with a travelling wave, which we now review. We consider a Euclidean world-sheet with complex coordinate $z = \exp(\tau - i\sigma)$ such that $\tau \in \mathbb{R}$ and $\sigma \in [0, \pi]$. We first review the boundary conditions applicable for a D-brane wrapped only once around y and later account for higher wrapping numbers.

We begin with the following world-sheet action for the superstring coupled to a background gauge field A^μ on a D9-brane following [23,28]:

$$S = S_0 + S_1, \tag{3.1}$$

where S_0 and S_1 are the world-sheet bulk and boundary actions respectively,

$$S_0 = \frac{1}{2\pi\alpha'} \int_M d^2z \left(\partial X^\mu \bar\partial X_\mu + \psi^\mu \bar\partial \psi_\mu + \widetilde\psi^\mu \partial \widetilde\psi_\mu\right), \tag{3.2a}$$

$$S_1 = i \int_{\partial M} dz \Big(A_\mu(X)\left(\partial X^\mu + \bar\partial X^\mu\right) - \frac{1}{2}\left(\psi^\mu + \widetilde\psi^\mu\right) F_{\mu\nu} \left(\psi^\nu - \widetilde\psi^\nu\right)\Big) \tag{3.2b}$$

and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Abelian field strength. In the absence of a boundary the action S_0 would be invariant under the supersymmetry transformations

$$\delta X^\mu = \varepsilon\psi^\mu + \tilde\varepsilon\widetilde\psi^\mu, \qquad \delta\psi^\mu = -\varepsilon\partial X^\mu,$$
$$\delta\widetilde\psi^\mu = -\tilde\varepsilon\bar\partial X^\mu \tag{3.3}$$

however the presence of the boundary breaks the $\mathcal{N} = 2$ world-sheet supersymmetry to $\mathcal{N} = 1$ supersymmetry. When we include S_1, the total action $S_0 + S_1$ preserves $\mathcal{N} = 1$ supersymmetry only up to the boundary conditions [28], which we impose at $z = \bar z$. Defining

$$E_{\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}, \tag{3.4}$$

varying the above action yields the boundary conditions [28]

$$\left[E_{\mu\nu}\widetilde\psi^\nu = \eta E_{\nu\mu}\psi^\nu\right]_{z=\bar z}, \tag{3.5a}$$

$$\Big[E_{\mu\nu}\bar\partial X^\nu - E_{\nu\mu}\partial X^\nu - \eta E_{\nu\rho,\mu}\widetilde\psi^\nu\psi^\rho - E_{\mu\nu,\rho}\psi^\nu\psi^\rho + E_{\nu\mu,\rho}\widetilde\psi^\nu\widetilde\psi^\rho\Big]_{z=\bar z} = 0, \tag{3.5b}$$

where η takes the value 1 or -1 corresponding to the NS and R sectors respectively. By applying the supersymmetry transformations (3.3) to the action (3.1) and employing these boundary conditions, one finds that (3.1) is invariant under the $\mathcal{N} = 1$ supersymmetry generated by these transformations with the constraint $\varepsilon = \eta\tilde\varepsilon$.

For the systems under consideration the gauge field takes a plane-wave profile and so A^μ will be a function only of the bosonic field $V = (X^0 - X^9)$, where X^0 is the string coordinate along time and X^9 indicates the compact y direction. A physical gauge field can be written as $A^I(V)$, where we set to zero the light-cone components. Then the non-vanishing components of $E_{\mu\nu}$ take the form

$$E_{uv} = E_{vu} = -\frac{1}{2}, \qquad E_{IJ} = \delta_{IJ},$$
$$E_{Iv} = -E_{vI} = \dot f_I(V), \tag{3.6}$$

where we have defined $f_I = -2\pi\alpha' A_I$.

We can rewrite the fields appearing (3.5a) and (3.5b) in modes by using the expansions

$$X^\mu(z, \bar z) = x^\mu - i\sqrt{\frac{\alpha'}{2}}\alpha_0^\mu \ln z - i\sqrt{\frac{\alpha'}{2}}\widetilde\alpha_0^\mu \ln\bar z + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m} + \frac{\widetilde\alpha_m^\mu}{\bar z^m}\right), \tag{3.7}$$

$$\psi^\mu(z) = \sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbb{Z}+\nu}\frac{\psi_r^\mu}{z^{r+\frac12}}, \qquad \widetilde\psi^\mu(\bar z) = \sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbb{Z}+\nu}\frac{\widetilde\psi_r^\mu}{\bar z^{r+\frac12}}, \tag{3.8}$$

where $\nu = 0$ and $\frac12$ for R and NS respectively. Note however that in our case the presence of a non-constant field strength $F_{\mu\nu}$ makes the boundary conditions nonlinear in the oscillators. We will see that, for the amplitudes in which we are interested, only the linear terms contribute.

As usual, we can change from the open string picture to the closed string picture, and derive the boundary conditions describing a closed string emitted or absorbed by the D-brane. This has the effect of

$$\alpha_n^\mu \to -\alpha_{-n}^\mu, \qquad \psi_r^\mu \to i\psi_{-r}^\mu, \quad \forall\mu, n, r. \tag{3.9}$$

We can then obtain the boundary conditions for a lower dimensional D-brane by performing a series of T-dualities; after these transformations, the components of f along the dualised coordinates describe the profile of the brane. We perform four or eight T-dualities in order to obtain the boundary conditions appropriate for a D5 or a D1-brane, for instance in order to move from the D9 frame to the D5–P frame we T-dualise along each x^i which sends

$$\widetilde\alpha_n^i \to -\widetilde\alpha_n^i, \qquad \widetilde\psi_r^i \to -\widetilde\psi_r^i. \tag{3.10}$$

By following the procedure outlined above, we can summarise the boundary conditions for the closed string oscillators as follows

$$\widetilde\psi_r^\mu = i\eta R^\mu{}_\nu\psi_{-r}^\nu + \cdots, \qquad \widetilde\alpha_n^\mu = -R^\mu{}_\nu\alpha_{-n}^\nu + \cdots, \tag{3.11}$$

where '…' indicates that we ignore terms which are higher than linear order in the oscillator modes. We shall justify this below (3.28). The reflection matrix R is obtained from (3.5a) and (3.5b) by performing the transformations (3.9) and (3.10) and replacing V by its zero-mode v:

$$R^\mu{}_\nu(v) = T^\mu{}_\rho\left(E^{-1}\right)^{\rho\sigma}E_{\nu\sigma}, \tag{3.12}$$

where the matrix T performs the T-duality (3.10), i.e. it is diagonal with values -1 in the x^i directions and 1 otherwise. R has the lowered-index form

$$R_{\mu\nu}(v) = \eta_{\mu\rho}R^\rho{}_\nu(v) = \begin{pmatrix} -2|\dot f(v)|^2 & -\frac12 & 2\dot f^i(v) & 2\dot f^a(v) \\ -\frac12 & 0 & 0 & 0 \\ 2\dot f^i(v) & 0 & -\mathbb{1} & 0 \\ -2\dot f^a(v) & 0 & 0 & \mathbb{1} \end{pmatrix}. \tag{3.13}$$

We refer the reader to [22–24] for a detailed discussion of the boundary state describing a D-brane with a travelling wave. For our purposes it is sufficient to know the linearized boundary conditions for the non-zero modes (3.11) that the boundary state must satisfy, and to construct explicitly only the zero-mode structure of the boundary state. Addressing firstly the bosonic sector, the boundary conditions on the zero modes are

$$p_v + \dot f^i(v) p_i = 0, \qquad p_u = 0, \qquad p_a = 0,$$
$$x^i = f^i(v), \tag{3.14}$$

where the first three equations follow directly from (3.11) and the fourth equation must be included to account for the T-duality transformations. The first equation in (3.14) may be represented as $i\frac{\partial}{\partial v} = \dot f^i(v) p_i$ and similarly the last constraint may be represented as $i\frac{\partial}{\partial p_i} = f^i(v)$. Then the boundary state zero-mode structure in the t, y and x^i direction is

$$\int dv\, du \int \frac{d^4 p_i}{(2\pi)^4} e^{-ip_i f^i(v)} |p_i\rangle |u\rangle |v\rangle. \tag{3.15}$$

So far we have essentially discussed a D-brane with a travelling wave in a noncompact space; we next generalise this description to the case of compact y and higher wrapping number. One may view a D-brane wrapped n_w times along the y-direction as a collection of n_w different D-brane strands with a non-trivial holonomy gluing these strands together. This approach was developed in [29,30] for the case of branes with a constant magnetic field.

In the presence of a null travelling wave with arbitrary profile $f(V)$, the individual boundary states of each strand will differ in their oscillator part and not just in their zero-mode part described above. However, we are interested in the emission of massless closed string states, which have zero momentum and winding along all compact directions. In this sector the full boundary state is simply the sum of the boundary states for each constituent, along with the condition that the value of the function f at the end of one strand must equal the value of f at the beginning of the following strand. We label the strands of the wrapped D-brane with the integer s; then restricting to the sector of closed strings with trivial winding (m) and Kaluza–Klein momentum (k), the boundary state takes the following form:

$$|D5; P\rangle^{k,m=0} = -\frac{\kappa \tau_5}{2} \sum_{s=1}^{n_w} \int du \int_0^{2\pi R} dv \int \frac{d^4 p_i}{(2\pi)^4} \times e^{-ip_i f^i_{(s)}(v)} |p_i\rangle |u\rangle |v\rangle |D5; f_{(s)}\rangle^{k,m=0}_{X,\psi}, \tag{3.16}$$

where $\tau_5 = [(2\pi\sqrt{\alpha'})^5 \sqrt{\alpha'} g_s]^{-1}$ is the physical tension of a D5-brane. We have written explicitly only the bosonic zero-modes along t, y and the x^i directions and we denote by $|D5; f_{(s)}\rangle^{k,m=0}_{X,\psi}$ the remaining part of the boundary state. The range of integration over $v = t - y$ follows from the periodicity condition of the space–time coordinate y.

We next address the fermion zero modes in the R-R sector. Letting $A, B, \ldots$ be 32-dimensional indices for spinors in ten dimensions,[1] and letting $|A\rangle|\widetilde{B}\rangle$ denote the ground state for the Ramond fields $\psi^\mu(z)$ and $\widetilde{\psi}^\mu(\bar z)$ respectively, the R–R zero mode boundary state in the $(-\frac{1}{2}, -\frac{3}{2})$ picture (before the GSO projection) takes the form

$$|D5; P\rangle^{(\eta)}_{\psi,0} = \mathcal{M}^{(\eta)}_{AB} |A\rangle_{-\frac{1}{2}} |\widetilde{B}\rangle_{-\frac{3}{2}} \tag{3.17}$$

where $\mathcal{M}$ satisfies the following equation [3],

$$\Gamma_{11} \mathcal{M} \Gamma^\mu - i\eta R^\mu{}_\nu \big(\Gamma^\nu\big)^T \mathcal{M} = 0. \tag{3.18}$$

A solution to this equation for the case of our reflection matrix R (3.13) is given by[2]

$$M = iC\left(\frac{1}{2}\Gamma^{vu} + \dot f^I(v)\Gamma^{Iv}\right)\Gamma^{5678}\left(\frac{\mathbb{1} - i\eta\Gamma_{11}}{1 - i\eta}\right). \tag{3.19}$$

where C is the charge conjugation matrix. The GSO projection has the effect of

$$|D5; P\rangle_{\psi,0} = \frac{1}{2}\big(|D5; P\rangle^{(1)}_{\psi,0} + |D5; P\rangle^{(-1)}_{\psi,0}\big) \tag{3.20}$$

and so the zero mode part of the D5–P R–R boundary state for the strand with profile $f_{(s)}$ is

$$|D5; f_{(s)}\rangle_{\psi,0} = i\left[C\left(\frac{1}{2}\Gamma^{vu} + \dot f^I_{(s)}(v)\Gamma^{Iv}\right)\Gamma^{5678}\frac{1 + \Gamma_{11}}{2}\right] \times |A\rangle_{-\frac{1}{2}} |\widetilde{B}\rangle_{-\frac{3}{2}} \tag{3.21}$$

which we can insert into the relevant part of the boundary state (3.16).

3.2. Disk amplitudes for the classical fields

We now calculate the fields sourced by the D5–P bound state by computing the disk one-point functions for emission of a massless state, starting with the NS–NS fields. Since the states are massless they have non-zero momentum only in the four noncompact directions of the $\mathbb{R}^4$, i.e. they have spacelike momentum (see also [3]). The NS–NS one-point function thus takes the form

$$\mathcal{A}^{(\eta)}_{\mathrm{NS}}(k) \equiv \langle p_i = k_i | \langle p_v = 0 | \langle p_u = 0 | \langle n_a = 0 | \times \mathcal{G}_{\mu\nu} \psi^\mu_{\frac{1}{2}} \widetilde{\psi}^\nu_{\frac{1}{2}} |D5; P\rangle^{k,m=0} \tag{3.22}$$

where for an S^1 direction with radius R we normalize the momentum eigenstates as $\langle n|m\rangle = 2\pi R\delta_{nm}$ and the position eigenstates as $\langle x|y\rangle = \delta(x-y)$. In terms of canonically normalized fields, $\mathcal{G}_{\mu\nu}$ is given by

$$\mathcal{G}_{\mu\nu} = \hat h_{\mu\nu} + \frac{1}{\sqrt{2}}\hat b_{\mu\nu} + \frac{\phi}{2\sqrt{2}}(\eta_{\mu\nu} - k_\mu l_\nu - k_\nu l_\mu), \tag{3.23}$$

where k_μ and l_ν are mutually orthogonal null vectors. The contribution to the zero mode part of the amplitude from a single strand with profile $f_{(s)}(v)$ is

$$V_4 V_u \frac{\kappa \tau_5}{2} \int_0^{2\pi R} dv\, e^{-ik_i f^i_{(s)}(v)}, \tag{3.24}$$

where V_u represents the infinite volume of the D-brane in the u direction. Since we have used a delocalised probe ($p_v = 0$), the string amplitude contains an integral over the length of the strand of the D-brane. In the classical limit n_w is very large, the typical wavelength of the profile is much bigger than R, and so f is almost constant over each strand [10,11]. The contribution to the value of each supergravity field is thus (3.24) divided by the volume of the strand:

$$\mathcal{A}^{(s)}_0(k) = \frac{\kappa \tau_5}{2} \frac{1}{2\pi R} \int_0^{2\pi R} dv\, e^{-ik_i f^i_{(s)}(v)}. \tag{3.25}$$

The contribution from the n_w different strands of the brane is therefore

[1] For the spinors and the charge conjugation matrix, we use the conventions of [3].

[2] The overall phase of M is a matter of convention; see also [5].

$$\mathcal{A}_0(k) = \frac{\kappa\tau_5}{2}\frac{1}{2\pi R}\sum_{s=1}^{n_w}\int_0^{2\pi R} dv\, e^{-ik_i f^i_{(s)}(v)}, \tag{3.26}$$

and we combine the integrals over each strand to give the integral over the full world-volume coordinate $\hat{v}$, giving

$$\mathcal{A}_0(k) = \frac{\kappa\tau_5}{2}\frac{n_w}{L_T}\int_0^{L_T} d\hat{v}\, e^{-ik_i f^i(\hat{v})}. \tag{3.27}$$

Adding in the non-zero modes, the coupling of the boundary state to the NS–NS fields is

$$\mathcal{A}^{(\eta)}_{\mathrm{NS}}(k) = -i\eta\frac{\kappa\tau_5 n_w}{2L_T}\int_0^{L_T} d\hat{v}\, e^{-ik_i f^i(\hat{v})}\mathcal{G}_{\mu\nu}R^{\nu\mu}(\hat{v}) \tag{3.28}$$

where $R(\hat{v})$ is the obvious strand-by-strand extension of the reflection matrix (3.13).

We can now observe why we were justified in ignoring terms higher than linear order in the oscillator boundary conditions (3.11). To arrive at the above result we substitute $\widetilde{\psi}^v_{\frac{1}{2}}$ for an expression involving only creation modes using (3.11), and only the linear term can contract with the remaining annihilation mode to give a non-zero result. A similar argument holds for the R–R amplitude.

The GSO projection has the effect of

$$\mathcal{A}_{\mathrm{NS}}(k) = \frac{1}{2}\big(\mathcal{A}^{(1)}_{\mathrm{NS}}(k) - \mathcal{A}^{(-1)}_{\mathrm{NS}}(k)\big) \tag{3.29}$$

and we read off the canonically normalized fields of interest via

$$\hat{h}_{vi}(k) = \frac{1}{2}\frac{\delta\mathcal{A}_{\mathrm{NS}}}{\delta\hat{h}^{vi}}, \qquad \hat{h}_{vv}(k) = \frac{\delta\mathcal{A}_{\mathrm{NS}}}{\delta\hat{h}^{vv}},$$
$$\hat{b}_{va}(k) = \frac{\delta\mathcal{A}_{\mathrm{NS}}}{\delta\hat{b}^{va}}. \tag{3.30}$$

The space–time configuration associated with a closed string emission amplitude is obtained by multiplying the derivative of the amplitude with respect to the closed string field by a free propagator and taking the Fourier transform [3]. In general for a field $a_{\mu_1\ldots\mu_n}$ we have

$$a_{\mu_1\ldots\mu_n}(x) = \int\frac{d^4k}{(2\pi)^4}\left(-\frac{i}{k^2}\right)a_{\mu_1\ldots\mu_n}(k)e^{ikx}, \tag{3.31}$$

with $a_{\mu_1\ldots\mu_n}(k)$ given in terms of derivatives of $\mathcal{A}$ as in(3.30). Using the identity

$$\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik^i(x^i-f^i)}}{k^2} = \frac{1}{4\pi^2}\frac{1}{|x^i-f^i|^2} \tag{3.32}$$

and the relation

$$Q_5 = \frac{2\kappa^2\tau_5 n_w}{4\pi^2}, \tag{3.33}$$

we obtain

$$\hat{h}_{vi} = \frac{Q_5}{2\kappa L_T}\int_0^{L_T}\frac{-\dot{f}_i d\hat{v}}{|x^i-f^i|^2}, \qquad \hat{h}_{vv} = \frac{Q_5}{2\kappa L_T}\int_0^{L_T}\frac{|\dot{f}|^2 d\hat{v}}{|x^i-f^i|^2},$$
$$\hat{b}_{va} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T}\frac{\dot{f}_a d\hat{v}}{|x^i-f^i|^2}$$

in agreement with (2.7).

We next calculate the coupling between the R–R zero mode boundary state and the on-shell R–R potential state [3–5]:

$$\langle\hat{C}_{(n)}| = {}_{-\frac{1}{2}}\!\left\langle\widetilde{B},\frac{k}{2}\right|{}_{-\frac{3}{2}}\!\left\langle A,\frac{k}{2}\right|\left[C\Gamma^{\mu_1\ldots\mu_n}\frac{\mathbb{1}-\Gamma_{11}}{2}\right]_{AB}$$
$$\times\frac{(-1)^n}{4\sqrt{2}n!}\hat{C}_{\mu_1\ldots\mu_n}, \tag{3.34}$$

where the numerical factor contains an extra factor of $\frac{1}{2}$ to account for the fact that we are not using the full superghost expression. Using the fact (see e.g. [4]) that

$$\big(\langle A|\langle\tilde{B}|\big)\big(|D\rangle|\tilde{E}\rangle\big) = -\langle A|D\rangle\langle\tilde{B}|\tilde{E}\rangle = -\big(C^{-1}\big)^{AD}\big(C^{-1}\big)^{BE}, \tag{3.35}$$

we find the coupling of the R–R potential to the (already GSO projected) boundary state for an individual strand (3.21) to be

$$\mathcal{A}^{(s)}_{\mathrm{R},\psi} = \langle\hat{C}_{(n)}|D5;f_{(s)}\rangle_{\psi,0}$$
$$= \frac{-i}{4\sqrt{2}n!}\mathrm{tr}\left[\Gamma_{\mu_n\cdots\mu_1}\left(\frac{1}{2}\Gamma^{vu}+\dot{f}^I_{(s)}(v)\Gamma^{Iv}\right)\right.$$
$$\left.\times\,\Gamma^{5678}\frac{1+\Gamma_{11}}{2}\right]_{AB}\hat{C}^{\mu_1\ldots\mu_n}. \tag{3.36}$$

This then combines with the bosonic zero mode part of the amplitude $\mathcal{A}^{(s)}_0$ given in (3.25) and we sum over strands to obtain the full R–R amplitude $\mathcal{A}_{\mathrm{R}}$. We then extract the gauge field profile via

$$\hat{C}^{(n)}_{\mu_1\ldots\mu_n}(k) = \frac{\delta\mathcal{A}_{\mathrm{R}}}{\delta\hat{C}^{(n)\mu_1\ldots\mu_n}} \quad (\mu_1<\mu_2<\cdots<\mu_n), \tag{3.37}$$

and as for the NS–NS calculation we insert the propagator and perform the Fourier transform. The fields which are non-trivial only in the presence of a travelling wave are then

$$\hat{C}^{(4)}_{vbcd} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T} d\hat{v}\,\frac{\dot{f}_a\epsilon_{abcd}}{|x_i-f_i(\hat{v})|^2},$$
$$\hat{C}^{(6)}_{vi5678} = \frac{Q_5}{\sqrt{2}\kappa L_T}\int_0^{L_T} d\hat{v}\,\frac{\dot{f}_i}{|x_i-f_i(\hat{v})|^2} \tag{3.38}$$

which agrees with (2.7). This completes the link between the microscopic and macroscopic descriptions of a D5-brane with a travelling wave.

To conclude, we have shown how to derive the supergravity fields sourced by D-brane/momentum bound states from disk amplitudes, describing in detail the D5–P frame calculation. In the D1–P frame, the computation proceeds exactly as for the $\mathbb{R}^4$ directions of the D5–P system and the results may be readily obtained by changing the index $i\to I$ and the charge $Q_5\to Q_1$ where appropriate. We have seen that our calculations reproduce the full form of the harmonic functions in the known supergravity solutions, which was not possible in the analogous calculation in the D1–D5 duality frame [7]. This is due to the fact that in the Dp-momentum duality frames the profile function parametrizing the solutions arises as a condensate of massless open strings related to the physical shape of the D-brane, which we were able to treat exactly using the boundary state formalism. We hope that combining this result with the analysis of the D1–D5 bound state will provide a way to study the three-charge D1–D5–P system using string amplitudes. Work in this direction is in progress.

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References

[1] M.R. Garousi, R.C. Myers, Nucl. Phys. B 475 (1996) 193, hep-th/9603194.
[2] A. Hashimoto, I.R. Klebanov, Nucl. Phys. B (Proc. Suppl.) 55B (1997) 118, hep-th/9611214.
[3] P. Di Vecchia, et al., Nucl. Phys. B 507 (1997) 259, hep-th/9707068.
[4] M. Billo, et al., Nucl. Phys. B 526 (1998) 199, hep-th/9802088.
[5] P. Di Vecchia, M. Frau, A. Lerda, A. Liccardo, Nucl. Phys. B 565 (2000) 397, hep-th/9906214.
[6] P. Di Vecchia, A. Liccardo, NATO Adv. Study Inst. Ser. C. Math. Phys. Sci. 556 (2000) 1, hep-th/9912161.
[7] S. Giusto, J.F. Morales, R. Russo, JHEP 1003 (2010) 130, arXiv:0912.2270.
[8] A. Sen, Mod. Phys. Lett. A 10 (1995) 2081, hep-th/9504147.
[9] S.R. Das, S.D. Mathur, Phys. Lett. B 375 (1996) 103, hep-th/9601152.
[10] O. Lunin, S.D. Mathur, Nucl. Phys. B 610 (2001) 49, hep-th/0105136.
[11] O. Lunin, S.D. Mathur, Nucl. Phys. B 623 (2002) 342, hep-th/0109154.
[12] A. Dabholkar, Phys. Rev. Lett. 94 (2005) 241301, hep-th/0409148.
[13] S.D. Mathur, Fortsch. Phys. 53 (2005) 793, hep-th/0502050.
[14] K. Skenderis, M. Taylor, Phys. Rept. 467 (2008) 117, arXiv:0804.0552.
[15] A. Sen, Two charge system revisited: Small black holes or horizonless solutions? arXiv:0908.3402.
[16] B.D. Chowdhury, A. Virmani, Modave lectures on fuzzballs and emission from the D1–D5 System, arXiv:1001.1444.
[17] A. Strominger, C. Vafa, Phys. Lett. B 379 (1996) 99, hep-th/9601029.
[18] I. Bena, N.P. Warner, Lect. Notes Phys. 755 (2008) 1, hep-th/0701216.
[19] A. Dabholkar, J.P. Gauntlett, J.A. Harvey, D. Waldram, Nucl. Phys. B 474 (1996) 85, hep-th/9511053.
[20] C.G. Callan, J.M. Maldacena, A.W. Peet, Nucl. Phys. B 475 (1996) 645, hep-th/9510134.
[21] C. Bachas, Ann. Phys. 305 (2003) 286, hep-th/0212217.
[22] Y. Hikida, H. Takayanagi, T. Takayanagi, JHEP 0304 (2003) 032, hep-th/0303214.
[23] J.D. Blum, Phys. Rev. D 68 (2003) 086003, hep-th/0304173.
[24] C.P. Bachas, M.R. Gaberdiel, JHEP 0403 (2004) 015, hep-th/0310017.
[25] O. Lunin, S.D. Mathur, A. Saxena, Nucl. Phys. B 655 (2003) 185, hep-th/0211292.
[26] O. Lunin, J.M. Maldacena, L. Maoz, Gravity solutions for the D1–D5 system with angular momentum, hep-th/0212210.
[27] I. Kanitscheider, K. Skenderis, M. Taylor, JHEP 0706 (2007) 056, arXiv:0704.0690.
[28] P. Haggi-Mani, U. Lindstrom, M. Zabzine, Phys. Lett. B 483 (2000) 443, hep-th/0004061.
[29] D. Duo, R. Russo, S. Sciuto, JHEP 0712 (2007) 042, arXiv:0709.1805.
[30] P. Di Vecchia, A. Liccardo, R. Marotta, I. Pesando, F. Pezzella, JHEP 0711 (2007) 100, arXiv:0709.4149.